$S_e e^{-jk_{xe}L}$ and $S_o e^{-jk_{xo}L}$, L being the length of the layer. If S_e and S_o are the elements of an appropriately normalized scattering matrix, then the reciprocity of the structure implies that the scattering matrix is symmetrical, so that S_e is also the appropriate scattering coefficient from the even mode of the layer to the even distribution of Rayleigh waves on the substrates beyond the layer, and similarly for S_o. As a result, the Rayleigh wave fields beyond the "output" of the layer are represented by the vector

$$\frac{1}{2} S_e^2 e^{-jk_{xe}L} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{2} S_o^2 e^{-jk_{xo}L} \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \frac{1}{2} S_e^2 e^{-jk_{xe}L} \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{S_o^2}{S_e^2} e^{-j(k_{xo}-k_{xe})L} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right]. \quad \text{(A.2)}$$

If the "aperture" field at the cross section containing the beginning of the layer is approximated by the field of the incident Rayleigh wave, the scattering coefficients S_e and S_o must then be real quantities because these coefficients are essentially the projection of the real aperture (Rayleigh wave) field on the modes of the layer, which are likewise real in their transverse field distribution. In practice, bulk waves are generated at the input and output junctions and these will contribute to the aperture field, but their effect is probably small.

Within the above approximation, therefore, the optimum transfer of energy from the lower to upper substrates is seen from (A.2) to occur when

$$(k_{xo} - k_{xe})L = \pi.$$

The ratio of energy on the upper and lower substrates is then given by:

$$\frac{P_u}{P_l} = \frac{|1 + S_o^2/S_e^2|^2}{|1 - S_o^2/S_e^2|^2}.$$

Complete transfer of energy can occur only if $S_o = S_e$, but this will not in general be the case. Nevertheless, the asymmetry of a single Rayleigh wave propagating on one substrate is such that neither of the layer modes is preferentially excited to any significant degree, so that one would expect S_o and S_e to be of comparable magnitude.

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Short Papers

Stripline Triplexer for Use in Narrow-Bandwidth Multichannel Filters

RALPH KIHLÉN

Abstract—**Design techniques and equivalent circuits are presented for constructing a printed-circuit narrow-bandwidth complementary triplexer filter. The techniques and circuits described allow the construction of contiguous-band multichannel filters using printed circuits with no shorted stubs.**

A unit was designed and constructed to give a three-percent relative bandwidth for each separate channel. The agreement between theory and experiment was in the range of measurement accuracy.

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Introduction

The design of a multichannel filter requires a network that will separate a given frequency band into N channels with minimum insertion loss and low VSWR at the input port. One way of solving this problem is to use cascaded-channel-separating units [1]–[3], i.e., diplexers, with constant input-port impedances. The advantage of this design is discussed by Matthaei and Cristal [1]. For each channel to be separated, one diplexer is needed. In order to reduce the number of separating units, the author has constructed a triplexer: a unit that separates out two contiguous channels. The total number of elements in a triplexer is the same as in two corresponding diplexers. However, the required space for a triplexer is less than that of two diplexers. The triplexer is a complementary or pseudo-complementary filter unit with constant input-port impedance and it can therefore be cascaded, as the diplexer, to obtain a multichannel filter system of various sizes without any interaction between the filter channels.

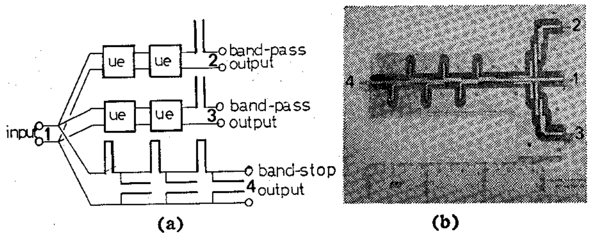

Fig. 1. A triplexer. (a) A prototype of a triplexer. (b) Photo of a stripline triplexer.

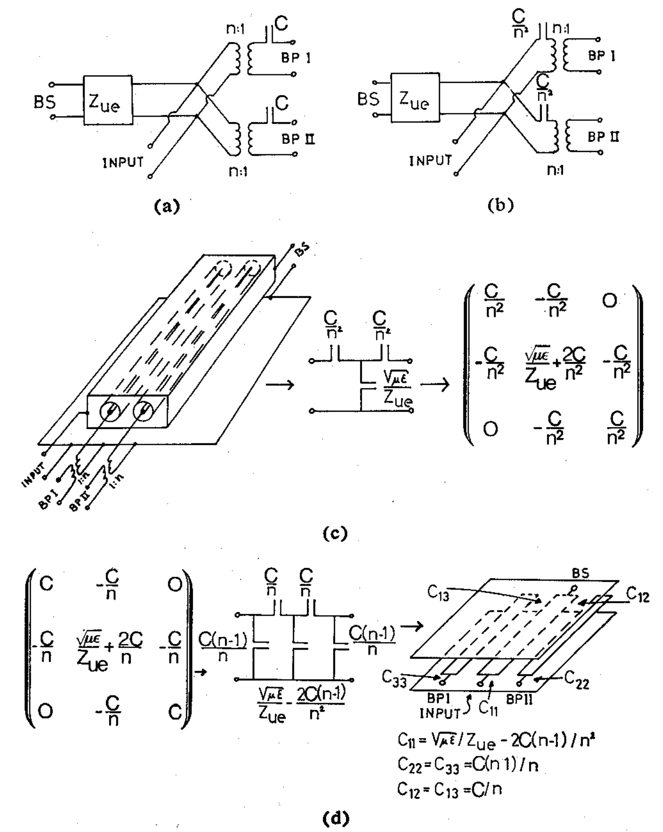

Fig. 2. The parallel interconnection section. (a) Possible interconnection section. (b) Transformed section. (c) Equivalent interconnection section, its static capacitance network, and static capacitance matrix. (d) Transformed static capacitance matrix eliminating the transformer, the static capacitance network, and its parallel-line configuration.

A photo of a narrow-bandwidth stripline triplexer is shown in Fig. 1(b) and its stub equivalent in Fig. 1(a). The filter consists of two bandpass filters and one bandstop filter connected in parallel. A diplexer similar to the triplexer has been described by Wenzel [2].

Filter Synthesis and Interconnection Section

A triplexer with maximally flat filter characteristics is to be designed. The filters in the triplexer do not have the same center frequencies as they do in the case of a diplexer. It is therefore impossible to design a triplexer with maximally flat characteristics and exactly constant input-port impedance. The synthesis and design of the two bandpass filters is outlined by Wenzel [2].

For the narrow-bandwidth case, it is possible to design the bandstop filter from a singly terminated low-pass LC prototype filter [1] with the number of elements double that of a bandpass filter to get a pseudocomplementary triplexer with reasonable low VSWR at the input port. For a triplexer with a 3-percent bandwidth for the bandpass filters, the maximum VSWR is calculated to be 1.058.

The three filters are connected in parallel. A possible connection is shown in Fig. 2(a). The unit element (Z_{ue}) is part of the bandstop filter, and one of the transformers ($1:n$) and one of the series stubs (C) are parts of one of the bandpass filters. An equivalent parallel-line net, its static capacitance network, and its static matrix [4] are shown in Fig. 2(c). By multiplying the first and last row and column in the matrix with n, the transformers are eliminated without changing the performance of the network. Fig. 2(d) shows the transformed matrix, its static capacitance network, and an equivalent parallel-line configuration.

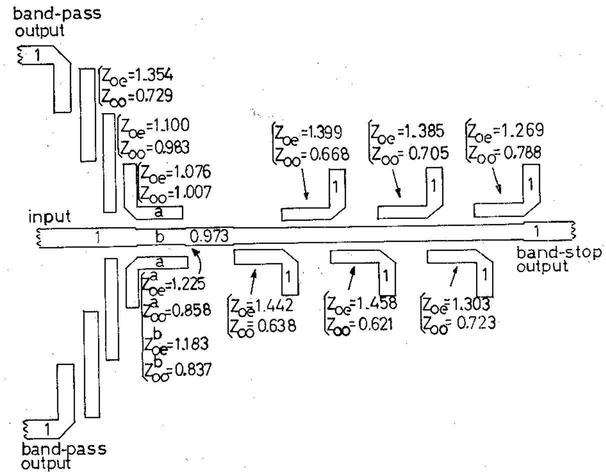

Fig. 3. Normalized impedances of the stripline triplexer.

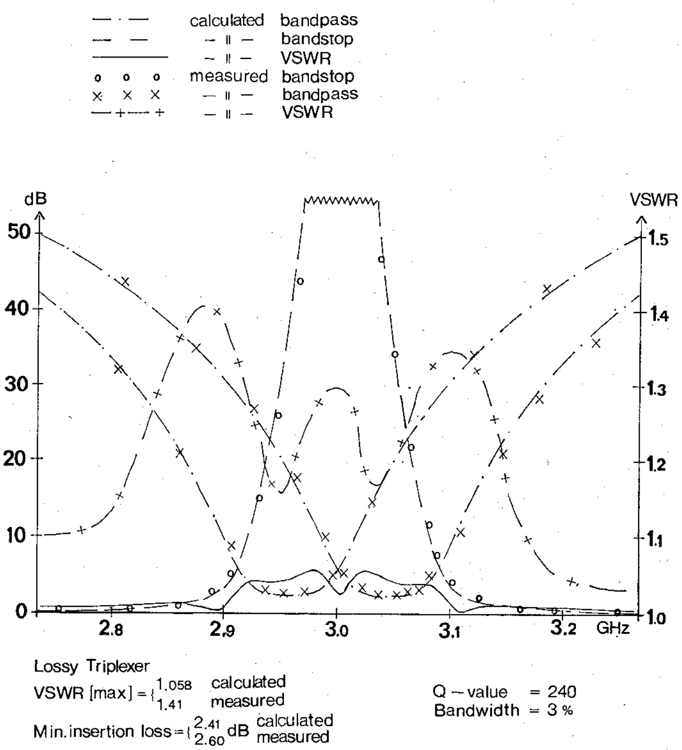

Fig. 4. Calculated and measured characteristics of the 3-percent bandwidth triplexer with the center frequency at 3.0 GHz.

Experimental Results

A maximally flat triplexer with a relative bandwidth for the bandpass filters of 3 percent was designed. The bandpass filters had three nonredundant elements each. The even- and odd-mode normalized characteristic impedances for a stripline triplexer shown in Fig. 3 are those obtained by the synthesis procedure outlined. Because of the redundant elements in the filters, one has considerable freedom in choosing the impedances in the bandpass filters. The physical dimensions are calculated from the even- and odd-mode impedances [5], [6].

A photograph of an experimental printed circuit in Rexolite 2200 for a triplexer is given in Fig. 1(b) and the test results in Fig. 4. The computed filter characteristics in Fig. 4 were obtained by assuming that all the elements of the triplexer have the same unloaded Q value as a stripline of 50 Ω in a Rexolite 2200 circuit board. No final adjustment of the dimensions was required to obtain this response. The resonator length reduction factor was 2.50 percent of a quarter of a wavelength at 3.0 GHz, as was calculated by Lagerlöf [7]. The electrical length of the corner was measured and found to be 9.62 percent of a quarter wavelength at the same frequency. The alignment of the center frequencies of the filters was of great importance. Care had to be taken in the photoetch process to get a filter requiring no final adjustment. Insertion loss at frequencies outside the crossover region was very low due to the loose coupling of the bandstop filter resonators.

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Implementation of Conservation-of-Energy Condition in Small Aperture and Small Obstacle Theory

CHUNG-LI REN

***Abstract*—In a previous paper, Felsen and Kahn showed that the scattering matrix of small apertures and obstacles in multimode waveguide regions is conveniently calculated for general lossless structures, but observed that the scattering matrix does not satisfy the conservation-of-energy requirement. It is also to be noted that the scattering parameters could become much larger than unity or even infinite for frequencies near or at the cutoff of the coupled modes. A method is presented in this correspondence to implement the lossless condition so that the resultant scattering matrix satisfies the conservation-of-energy requirement and, consequently, can be represented as a lossless equivalent circuit for all frequencies. The corresponding impedance, admittance, and transfer matrices for general lossless symmetrical structures are given in compact form directly in terms of the scattering parameters.**

I. Introduction

The design of waveguide components requires the availability of specific discontinuity structures with known transmission and reflection properties. A rigorous theoretical analysis of these waveguide discontinuities is very often quite involved and, in practice, its solution usually becomes tractable only with the imposition of judicious assumptions. One such assumption is that the apertures and the obstacles are small and the solutions may be evaluated easily in the lowest order of approximation, which is generally known as the small aperture and small obstacle theory [1]. The application of small aperture and small obstacle theory to discontinuities in multimode waveguide regions becomes particularly attractive in view of the fact that such design information is generally unavailable in the literature, whether in the form of theoretical calculation or measurements. The design of millimeter wave waveguide components, such as filters and couplers involving multimode propagation, is such an example.

However, the scattering matrix of a lossless waveguide discontinuity calculated from small aperture and small obstacle theory is not unitary and, hence, in violation of the conservation of energy [1]. Thus the impedance or admittance matrices, when converted from the scattering matrix, always contain real parts. In addition, the scattering parameters become very large or even infinite for frequencies at which certain modes are near or at cutoff. Therefore, meaningful equivalent circuits cannot be derived from these scattering matrices. In this correspondence, a technique is proposed to implement the conservation-of-energy condition so that the modified scattering matrices satisfy this condition. The corresponding impedance, admittance, and transfer matrices are derived in compact form directly in terms of the scattering parameters.

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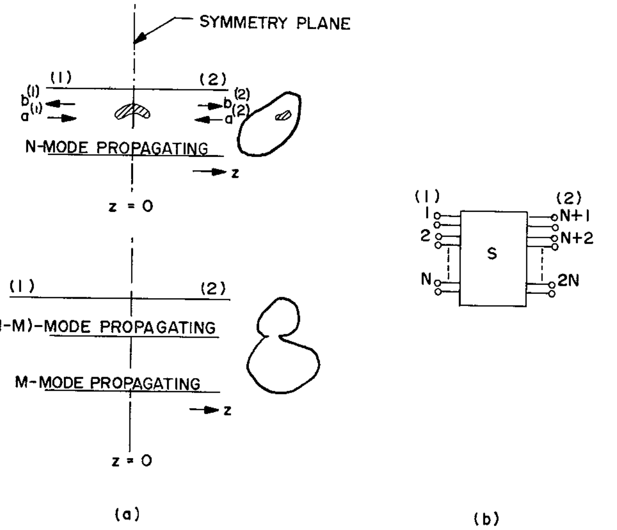

Fig. 1.

II. Small Aperture and Small Obstacle Scattering Formulation of Lossless Symmetrical Discontinuities in Multimode Waveguides

Consider the configuration in Fig. 1 where either a perfectly conducting obstacle or an aperture is located in a waveguide or between several waveguides propagating N modes. For convenience in the derivation of the theory, the structures of Fig. 1 are assumed to be symmetric in the sense that there exists a transverse plane of bisection at $z=0$. Such structures are the most frequently encountered discontinuities in the waveguide component designs.

The small aperture and small obstacle formulation for the scattering coefficients of the structures in Fig. 1 is given in equations (10) and (33) of [1], which may be generalized and written in a matrix form shown in (2).

$$b = Sa \qquad a = \begin{pmatrix} a^{(1)} \\ \vdots \\ a^{(2)} \end{pmatrix} \qquad b = \begin{pmatrix} b^{(1)} \\ \vdots \\ b^{(2)} \end{pmatrix} \tag{1}$$

$$S = \left(\begin{array}{c|c} A_1 + A_2 & I + A_1 - A_2 \\ \hline I + A_1 - A_2 & A_1 + A_2 \end{array} \right)_{2N \times 2N} \tag{2}$$

where I is the $N \times N$ identity matrix. A_1 and A_2 are imaginary $N \times N$ submatrices, which are functions of the polarizabilities of the discontinuity and the electromagnetic fields of the modes that are coupled by the discontinuity. In the general case, both A_1 and A_2 are nonzero. However, either A_2 or A_1 is a zero matrix for structures that are either pure shunt or pure series, respectively. For example, when all modes are coupled only through their longitudinal magnetic field components and (or) their transverse electric field components, A_1 is nonzero, $A_2=0$, and the structure is pure shunt. In the dual case when only the longitudinal electric field and (or) the transverse magnetic fields are coupled, A_2 is nonzero and $A_1=0$. The structure becomes pure series. It is to be noted that the scattering matrix in (2) is not unitary and does not satisfy the conservation-of-energy requirement [1]. In the submatrices A_1 and A_2, certain elements may